The Skill Of Sixth Grade Students To Use Multiple Representations in Addition and Subtraction Operations in Fractions

**Abstract.** Current research aims to show the transition skills of sixth grade students between representations (numerical, model, number and verbal lines ) given in addition operations and reduction of fractions and to analyze the ability of students to develop each type of representation. Subject in this study were sixth grade students consisting of 22 students, 10 of whom were male and 12 of them were women. Data is collected through several student representations in fractional operation tests. According to the results of the study, it was found that the achievement of students in using different representations in addition operations in fractions was higher than the reduction operation. In addition, it turns out that students are more successful in transmitting representations from numerical to numerical, models, numerical models, and numerical-models compared to other transitions of the two types of operations. It was also found that students failed in other types of representations such as number lines and verbal representation transitions.

**Keywords:** Multiple representations; addition and subtraction of fractions; representational skills; transition between representations.

**1. Introduction**

Mathematics is a field that consists of the concept of ab stract, algorithm, and symbol without any connection with the real world, that is the definition of the person's behavior (eg Cramer, 2003). For these reasons , researchers emphasized the need to teach mathematics as an integrated concept and processing systems based on certain patterns and associations that exist in the real world (Nair & Pool, 1991; Resnick & Ford, 1981). This requirement led to a lengthy debate in terms of using the most appropriate representations da lam teaching and learning of mathematics. It has a complete understanding of mathematical concepts, expresses ideas and mathematical relationships between concepts (Duval, 2006; Goldin & Shteingold, 2001). , Using different representations in teaching mathematical concepts and making the transition between various forms of representation very important in terms of complete mathematics internationalization. (Kaput , Blanton, & Moreno, 2 008; Lesh, 1999; National Coun cil of Teachers of Mathematics [ NCTM], 2000).

Therefore , the use of representation has become an important topic in mathematics learning during the last three decades in the standard of school mathematics to develop the ability of students to use right representation. (Ministry of National Education [Ministry of National Education, 2013]; National Council of Teachers of Math Ematics [NCTM], 2000; Van de Walle, Karp, & Bay-Williams, 2010). But, the study focuses on the ability of students in it under the representation that shows that high school students have the knowledge and ability t idak sufficient to establish the most appropriate representations and change from one representation to another (Gagatsis & Elia, 2004; Neria & Amit, 2004). In this context, mathematics teachers need to consider and effectively use several representations of information in verbal, numerical, graphical or visual numerical forms , with the support of technological development , rather than just using intensive verbal and mathematical languages. Research on various representations in mathematics has shown that using multiple representations helps to help students better understand and improve their problem solving (Ainsworth, Bib by, & Wood, 1997; Akkuş-Çıkla, 2004; Moseley & Brenner, 1997; Sert , 2007). If not able to switch between different representations, it can be said that mathematics a cannot be understood at the conceptual level (Ainsworth, 1999; Van der Meij & De Jong, 2006), However, there is no detailed explanation about various representations on operations of addition and subtraction of fractions.

In the context of mathematics education, research around the world focuses more on students' understanding of fractions and fraction operations, this shows that fraction learning is complex and a difficult process . and the ability to change between many representations (Tunç-Pekkan, 2015). Looking at students' difficulties and mistakes about adding fractions , researchers suggest using some representations in teaching and learning explosive meaning in overcoming student difficulties and developing their fraction conceptual understanding ( Tunç- Pekkan, 2015).

The development of the way students think about representation in mathematics learning reveals the need for the use of various representations (Pape & Tchoshanov, 2001). On the other hand , the findings of this study are considered to contribute to the alternative use of pedagogical approaches to mathematics educators and researchers , and to investigate the effects of this approach on classroom settings.

**2. Research Methodology**

Types of research

In this study using a qualitative research approach.

Research subject

The subjects in this study were class VI students . this study uses a purposive sampling method. Initially 24 students, 12 men and 12 women but from the pre-evaluation results there were 2 students who did not answer the questions at all. Then they were not included, and finally the research subjects consisted of 22 students, 10 men and 12 women.

Research instrument

Data collection used in this study are:

1. Using a questionnaire containing the sex of students , age and math class notes .
2. Various representations in fractional operation tests.

For test trials, 30 test items were given consisting of 8 main questions and sub questions. To determine the validity and reliability of t es, three expert opinions drawn from the field of mathematics education. And the 30 question trial , a pre-test was given to 24 students.

Data analysis

The level of achievement of students in the transition between representations and correct answers according to questions. For example, if a question is given in the form of numbers , and a verbal explanation is sought for a solution, then the value has been reached there is a transition from numerical to verbal representation . Students' skills in making each representation are analyzed using the following criteria

Table. 1 Criteria for making representations

|  |  |  |  |
| --- | --- | --- | --- |
| Numerical | Number line | Verbal | Model |
| 1. Writing the mathematical of the representation
2. Denominator equalization if necessary
3. Expressing the given operation numerically
4. Performing the operation
 | 1. Drawing the number line
2. Placing the integers
3. Separating by equal units (determining the denominator)
4. Expressing fraction (showing numerator with arrows)
5. Performing the given operation
6. Showing the result (arrowing out)
 | 1. Expressing the given fractions (verbal) correctly
2. Identification of objects suitable for given fraction (wholes to be fragmented)
3. Performing the given operation in the text
4. To be able to express the process given at the root of the question
 | 1. Drawing a closed figure devided by the appropriate units for given fractions (determining the denominator)
2. Screening the requested part according to the given fractions (determining the numerator)
3. Denominator equalization if necessary
4. Performing the given action on figures
 |

**3.Findings**

The results of the study of the transition skills of sixth grade students in addition and subtraction fractions is given in Table 2. The table shows that students are more successful in numerical responses to questions given in the form of model representations (transitions of the kenumeric model) and cases where numerical expressions of verbal questions (verbal to numerical transition). Likewise, most students succeed in transitions that require to build models and numerics in addition operations. In addition, the performance of students in transition line numbers and verbal representation is relatively low (between 16% and 24 %)

Table 2. *Student's achievement rate in transiting between representations (%)*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| To:  | Model  | Numerical  | Number Line  | Verbal  |
| From  | +  | -  | +  | -  | +  | -  | +  | -  |
| Model  | 66  | 41  | 78  | 49  | 12  | 36  | 21  | 9  |
| Numerical  | 56  | 49  | 66  | 62  | 17  | 26  | 22  | 10  |
| Number Line  | 53  | 36  | 56  | 38  | 12  | 17  | 16  | 9  |
| Verbal  | 68  | 33  | 77  | 31  | 24  | 16  | NR  | NR  |

Note: NR: Transition note required

Results penelitain shown in Table 2 te rlihat that the status of student achievement in the reduction operation generally lower than the sum of operation. And it can be seen also , it turns out that most students succeed in questions that make up the model - numerical and numerical - models in reduction operations. In addition, it can be observed that in a positive transition in a reduction operation, students show very low performance, especially in transitions that require the creation of verbal representations.

*Problems faced in forming representations*

In this section, the problems encountered on the representation y ang d ianalisis and presented in accordance with predetermined criteria. The values given are the values obtained from the responses given kep no question as to form on each representation.

*Problems in forming model representations*

Table 3. *Issues encountered in model representation forming*

|  |  |  |  |
| --- | --- | --- | --- |
| Steps  | Addition  | Subtraction  | Total  |
| Determining the denominator  | 56  | 49  | 105  |
| Determining the numerator  | 44  | 56  | 100  |
| Equalizing denominators if necessary  | 31  | 53  | 84  |
| Fullfilling the operation  | 49  | 73  | 122  |
| Unanswered  | 19  | 48  | 67  |

Table 3 is list the difficulties faced by students at Repr esentasi operating model in addition and subtraction of fractions. The result showed that the difficulties students on se mua steps to be followed during the process p Setting up a national representation of the model and add fractions on the model. Students make more mistakes in the process of determining the denominator and completing the operation. Likewise students leave more questions unanswered in the reduction operation .



Figure 1. Denominator mistake in transition from model representation to model representation

Figure 1 indicates an mistake in determining the denominator of the summation operation. In accordance with Figure 1, students are asked to add up right fractions 2/12 and 4/12 given on the models. Students have drawn 8 units (denominators) and shaded or marked 6 units (numerator) in the model created in the answer and declare the solution as 6/8. When doing

addition operation in the model representation, students appear not to focus on the denominator on the final answer, still i only add the numerator or the marked area .

*Problems encountered in forming numerical representations*

In table 4 , the problems faced by students in forming numerical asi representations are presented according to the specified criteria . It can be seen from the error found in each step to be followed in the process creating a numerical representation while an error is found in doing fewer numerical operations than the fractional model operation . However, there are more questions that are not answered in numerical representation ( *f* = 86 ) than the first model ( *f* = 67 ).

When adding fractions with m enggunakan numerical representation, students are experiencing errors in steps m elakukan operation and write numeric representation. It can be seen that students leave the question in the abatement operation, more errors are found in the reduction operation phase .

Table 4. *Issues encountered in numerical* *representation forming* *(f)*

|  |  |  |  |
| --- | --- | --- | --- |
| Steps  | Addition  | Subtraction  | Total  |
| Writing the numerical expression of the representation  | 28  | 31  | 59  |
| Expressing the operation numerically  | 18  | 31  | 49  |
| Equalizing denominators if necessary  | 20  | 55 | 55  |  |
| Performing the operation  | 40  | 57  | 97  |
| Unanswered  | 28  | 58  | 86  |

Upset ahan often experienced by students in numerical representation, this can be seen in Figure 2 that the student is right in expressing fractions given in represen tation of the model can not be expressed in the form of a numerical representation. This can be observed from the operations performed by students, students subtract the first numerator from the second to find the numerator, and reduce the denominator of the denominator to calculate the denominator. He carried out the cutting operation by thinking of the denominator and numerator separately . In other words, the results are obtained by reducing the denial and denominator itself.



Figure 2. Performing operation mistake in transition to model representation to numerical representation

*The problems faced in forming a number line representation*

In table 5 presents the annoyance of the students made when doing the addition and subtraction of fractions in line b in the light. Students making a lot of errors in rare h-steps expressing fractions in a number line ( showing arrows), specifies the operation given in the number line and shows the result (with an arrow out), even though the error is found at each step in general. In addition, it can be observed that most students do not answer questions.

Table 5. *Issues encountered in number line* *modeling* *forming* *(f)*

|  |  |  |  |
| --- | --- | --- | --- |
| Steps  | Addition  | Subtraction  | Total  |
| Drawing number line  | 1  | 1  | 2  |
| Placing the integers  | 23  | 15  | 38  |
| Separating by equal units (determining the denominator)  | 56  | 46  | 102  |
| Expressing fractions (showing numerator with arrows)  | 106  | 76  | 182  |
| Determining the operation  | 89  | 89  | 178  |
| Showing the result (arrowing out)  | 124  | 91  | 215  |
| Unanswered  | 56  | 82  | 138  |



Figure 3. Separating into equal units (determining the denominator)

Figure 3 gives an example of one of the student's mistakes that is often done which in dividing the whole into the same part and identifying on the number line, which is placing the denominator. While the whole is needed to divide into twelve equal parts in the number line, it can be seen that twelve lines are drawn as a whole and divided into thirteen parts.

*Problems* *faced in verbal representation*

In table 6 , errors made by students when conducting summation operations and a reduction in verbal representation in fractions are identified. In general, errors are found in faithful steps, it is known that most mistakes are made by students in the steps of making verbal representations that match the scenario and expressing the operation given to the question correctly. It should be noted that the number of questions that were not answered in the operation of additional reduction in verbal verbal representation was rather high, and the number of unanswered questions was higher in the reduction operation compared to the addition operation .

Table 6. *Issues encountered in verbal* *representation forming* *(f)*

|  |  |  |  |
| --- | --- | --- | --- |
| Steps  | Addition  | Subtraction  | Total  |
| Expressing the fractions correctly  | 33  | 25  | 58  |
| Identification of objects suitable for given fractions (wholes to be fragmented)  | 26  | 34  | 60  |
| Creating a scenario that is suitable for given operation  | 37  | 61  | 98  |
| Being able to express the operation is given at the root of the question  | 85  | 83  | 168  |
| Unanswered  | 53  | 70  | 123  |



Figure 4. Mistake in scenario creation and question root determination in transition to model representation to verbal representation.

In figure 4 is an example of frustration in making a scenario and expressing the operation given to the root of the question in relation to the verbal representation of the operation of the representation of the given model . Students have prepared a scenario by expressing it mathematically, the fractions given and spending 2/12 of the money first, then 4/12 in the scenario. Students, while the screenplay problems, can not reveal the starting position clearly, can not fully express the sentence and also creating root questions such as "how much money he's spent?" Fig cement supposed to be "part of what money he had spent ? ".

**4. Conclusions and Discussion**

In this study, using multiple representations and trans formation of one another are sought by focusing on the representation of the model, the representation of the number line, the representation of verbal and numerical representation in the operations of addition and subtraction in fractions. According to the results of the study, it is known that the achievement of students in using different representations in fractional operations is higher in addition operations than in the reduction operation. In addition, it turns out that students are more successful in numerical-numerical, numerical-model, and numerical-model transitions compared to other transitions in both types of operations. This situation is parallel to various studies that reveal student achievement towards numerical representation and models (Herman, 2002, Pitts, 2003). It is also known that they fail in other types of representations such as number line representation and verbal representation . Students also state the difficulties experienced by students in verbal (Sert , 2007).

In forming the model representation, more errors are identified in the steps of determining the numerator and denominator in the addition operation. On the other hand, errors in fractional reduction operations occur more frequently in the steps of carrying out operations and determining the numerator. This indicates students who have experienced difficulty in establishing the concept of fractions.

In numeric representation, students are more challenged. In the n- deduction operation , and errors occur in the steps of performing operations and writing numeric expression of representation. Especially in fractions with different denominators, this determines that students encourage their natural habits to think of the denominator and numerator as separate numbers. Relevant problems have been widely described in the literature (Carpenter, Coburn , Reys, & W ilson, 1976 ).

In the studies they conducted , they also identified problems such as the inability of students to understand the specific quantity of numbers in fraction surgery and the inability to operate operations to the root of the question . The contents of the identified difficulties and the high number of difficulties identified in the complicated problem show that students are not dimmed into the concepts in fractions and addition and subtraction operations in fractions. These results support the conclusion that difficulties oriented to fraction operations cannot be evaluated reliably from the difficulties in learning fractions and their roots are based on the concept of development as shown by Charalambous, Delaney, Mhuire, Hsu and Mesa (2010).

Research also shows that teaching can inhibit fraction learning, especially when (1) fails to build students' initial knowledge (Mack, 2001), (2) emphasizes rote learning at the expense of conceptual understanding (Ball, 1993; Mack, 2001), (3) it introduces formal symbols and algorithms before familiarizing students with different aspects of fractions (Smith, 2002), or (4) emphasizing only one construct (whole parts) (Moss & Case, 1999). Book text, as one of the instruction tools, can contribute to adding or correcting these difficulties. In the framework used in this study these criteria are given detailed consideration.

The findings obtained from this study are considered important for those who guide educational policy and those who contribute to the preparation of the mathematics curriculum. Educational research studies are important to explain the development of education reforms and curricula prepared for this direction. In this context, it is estimated that institutions or individuals involved in this process will contribute to matching the findings obtained from this study with targeted output in education policies or curricula.

This is research carried out in the form of special case analysis. Qualitative research or further interventions that focus on teacher classroom practices, student preferences for representation and the ability to use representations may be useful to investigate and broaden the results obtained in this study. It is also intended that the tools developed to determine representation of the use of competencies or skills, perceptions or attitudes towards representation of students (or teachers) will contribute to quantitative and qualitative studies with the subject of "multiple representation and mathematical learning". The results showed that students experienced difficulties in verbal representation of addition operations and subtraction in fractions to develop problem solving skills in developing these skills, it is important to build relationships between real life and fraction situations (Abu-Elwan, 2002, Akay & Boz, 2008, Dickerson, 1999). In addition, this activity supports conceptual understanding of students (Akay, 2006; Crespo da n Sinclair, 2008; English, 2003; Stickles, 2006; TolukUçar, 2009). For this reason, the teacher must give students the opportunity to express themselves verbally and linguistically to improve their problem solving skills in the lesson and give students the opportunity to solve problems and problem solving methods.

Again, when verbal questions are asked, the part of the scenario must be cut in half and students must be asked to complete or part of the root of the question must be left blank, students must be obtained by revealing fractions .

Abstract fractions for middle school students. When the concept of fractions is explained only by verbal expression or numerical representation, students cannot understand these concepts which are abstract to them (Piaget, 1952). For this reason, the use of models and teaching materials that represent various forms of representation in teaching summarizes and subtraction in fractions is very important for the realization of meaningful learning. This study can be considered as a study that mainly highlights prospective teachers and teachers.

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